Equations for QR talk-AlgoDiff 2024

Lucas Roberts, rlucas7@vt.edu

1 Wide case

Let $\bar{\boldsymbol{Q}}_p = \bar{\boldsymbol{Q}} + \boldsymbol{Y}\bar{\boldsymbol{V}}^T$ For $\boldsymbol{A} \in \mathbb{R}^{r \times c}$ let $\boldsymbol{A} = \boldsymbol{QR}$ then $\boldsymbol{A} = [\boldsymbol{X}|\boldsymbol{Y}] = \boldsymbol{QR} = \boldsymbol{Q}[\boldsymbol{U}|\boldsymbol{V}]$. $\bar{\boldsymbol{A}} = [\left(\bar{\boldsymbol{Q}}_p + \boldsymbol{Q}\,copyltu(\boldsymbol{M})\right)\boldsymbol{U}^{-T}|\bar{\boldsymbol{Y}}]$, (eqn 3.3).
$\bar{\boldsymbol{A}} = [\boldsymbol{Q}\left(\bar{\boldsymbol{Q}}_p + \boldsymbol{P}_L \circ \left(\boldsymbol{U}\bar{\boldsymbol{U}}^T - \bar{\boldsymbol{U}}\boldsymbol{U}^T + \boldsymbol{Q}^T\bar{\boldsymbol{Q}}_p - \bar{\boldsymbol{Q}}_p^T\boldsymbol{Q}\right)\boldsymbol{U}^{-T}\right] + \left(\bar{\boldsymbol{Q}}_p - \boldsymbol{Q}\boldsymbol{Q}^T\bar{\boldsymbol{Q}}_p\right)\boldsymbol{U}^{-T}|\bar{\boldsymbol{Y}}]$.
In both equations $\bar{\boldsymbol{Y}} = \boldsymbol{Q}\bar{\boldsymbol{V}}$. Non-wide case is a special case of wide case with appropriate outer product of empty matrices.

2 Deep/Tall and square (Non-wide)

For $\boldsymbol{A} \in \mathbb{R}^{r \times c}$ let $\boldsymbol{A} = \boldsymbol{QR}$ then,
$\bar{\boldsymbol{A}} = \left(\bar{\boldsymbol{Q}} + \boldsymbol{Q}\,copyltu(\boldsymbol{M})\right)\boldsymbol{R}^{-T}$ with $\boldsymbol{M} = \boldsymbol{R}\bar{\boldsymbol{R}}^T - \bar{\boldsymbol{Q}}^T\boldsymbol{Q}$, (eqn 3.3)
$\bar{\boldsymbol{A}} = \boldsymbol{Q}\left[\bar{\boldsymbol{R}} + \boldsymbol{P}_L \circ \left(\boldsymbol{R}\bar{\boldsymbol{R}}^T - \bar{\boldsymbol{R}}\boldsymbol{R}^T + \boldsymbol{Q}^T\bar{\boldsymbol{Q}} - \bar{\boldsymbol{Q}}^T\boldsymbol{Q}\right)\boldsymbol{R}^{-T}\right] + \left(\bar{\boldsymbol{Q}} - \boldsymbol{Q}\boldsymbol{Q}^T\bar{\boldsymbol{Q}}\right)\boldsymbol{R}^{-T}$
(eqn 3.8)
(prior work from S. Walther)
$\boldsymbol{P}_L$ is a strictly lower tridiagonal matrix with all ones beneath the diagonal and zeroes along and above the main diagonal